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RESUME OF NOZZLE DAMPING THEORY

by

George L. Dehority
Aerothermochemistry Division
Research Department

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Nozzle damping theory was originally developed for liquid rockets. Typically, there is an injector at the head end of a cylindrical combustion chamber terminated by a nozzle, a geometry far simpler than that for a typical solid propellant rocket. The damping associated with a rocket nozzle, in many cases, reduces to evaluation of the acoustic admittance at the nozzle entrance plane. For small perturbations the sonic point in the nozzle is unique in that downstream of that point no wave energy will be reflected back to the chamber since the wave propagation velocity is less than or equal to the mean flow velocity. The analysis thus requires solving the flow field from the sonic point back to the nozzle entrance plane. Simpler treatments are available for axial modes when the convergence section of the nozzle is much shorter than the wave length and will be treated in a separate section.

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FOREWORD

The occurrence of acoustic oscillations in rocket motors is a very troublesome problem. The ultimate goal in motor stability theory is the ability to predict oscillations in the design stage and to know what measures may be taken to eliminate them or reduce their magnitude to an acceptable level.

Before any such overall stability prediction may be made it is first necessary to understand all of the important sources of acoustic driving and damping in a rocket motor. This report will deal with one aspect of the subject, namely nozzle damping. It is basically a summary of techniques commonly used to evaluate nozzle damping both theoretical and experimental. A reference list to much of the appropriate literature is provided.

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R.L. DERR
Head, Aerothermochemistry Division
Research Department
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INTRODUCTION

This report is a brief survey of rocket nozzle damping which can provide a major source of damping in solid propellant motors. In a stability analysis it would be very advantageous to be able to assess nozzle damping. Unfortunately the state of the art precludes a reliable evaluation for many types of solid propellant motors.

Nozzle damping theory was originally developed for liquid rockets. Typically, there is an injector at the head end of a cylindrical combustion chamber terminated by a nozzle, a geometry far simpler than that for a typical solid propellant rocket. The damping associated with a rocket nozzle, in many cases, reduces to evaluation of the acoustic admittance at the nozzle entrance plane. For small perturbations the sonic point in the nozzle is unique in that downstream of that point no wave energy will be reflected back to the chamber since the wave propagation velocity is less than or equal to the mean flow velocity. The analysis thus requires solving the flow field from the sonic point back to the nozzle entrance plane. Simpler treatments are available for axial modes when the convergence section of the nozzle is much shorter than the wave length and will be treated in a separate section.

RESUME OF FUNDAMENTAL MODELS

A model for three-dimensional waves in a nozzle with a one-dimensional mean flow is derived in Ref. 1. It should be noted that the model is for a nozzle with a cylindrical cross section.* The one-dimensional restriction on mean flow implies that the nozzle entrance section is slowly converging. Solutions for conical nozzles joining a cylindrical tube are presen[illegible] in the form of admittance data at the nozzle entrance. The authors point out that one integration will result in many solutions since each integration step represents a different motor nozzle geometry.

*The case of a two-dimensional nozzle is also treated but will not be considered here and the discussion will be limited to small perturbations.

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The same basic analysis was presented in Ref. 2. It differs mainly in the fact that the authors were able to show that an assumption of irrotational flow had only a small effect on the calculated results. It also expanded the analysis to include the solution for growing (or decaying) oscillations as well as for steady-state oscillations. The report includes a computer program for solving the flow field for the convergent section of a supersonic nozzle. The result of primary interest is the complex admittance, the real part of which is used to calculate nozzle damping.

EXPERIMENTAL DETERMINATION OF NOZZLE ADMITTANCE

Crocco, Monti, and Grey (Ref. 3) determined nozzle admittance for axial modes by direct measurement of the perturbation velocity and pressure at the nozzle plane. The half power band width technique for axial modes was used in Ref. 4 to measure nozzle damping. Reference 5 is useful in analyzing the resulting measurements and relating the damping results to the admittance. A third technique is the impedance tube method. This was the technique used in Ref. 6 to experimentally determine three-dimensional nozzle admittances for a motor geometry typical of liquid rockets. The experimental results were found to be in good agreement with the theoretical results of Ref. 2.

The resultant expressions for the real and imaginary part of nozzle admittance taken from Ref. 6 are

$$y_r = \frac{\dfrac{S[S^2 - S_{mn}^2\,(1 - M^2)]^{1/2}\,\tanh(\pi\alpha)\,\sec^2(\pi\beta)}{\tanh^2(\pi\alpha) + \tan^2(\pi\beta)} - S_{mn}^2 M}{S^2 + (S_{mn}M)^2} \tag{1}$$

and

$$y_i = \frac{S[S^2 - S_{mn}^2\,(1 - M^2)]^{1/2}\,\tan(\pi\beta)\,\mathrm{sech}^2(\pi\alpha)}{[S^2 + (S_{mn}M)^2][\tanh^2(\pi\alpha) + \tan^2(\pi\beta)]} \tag{2}$$

and for axial modes ($S_{mn} = 0$)

$$y_r = \frac{\tanh(\pi\alpha)\,\sec^2(\pi\beta)}{\tanh^2(\pi\alpha) + \tan^2(\pi\beta)} \tag{3}$$

and

$$y_i = \frac{\tan(\pi\beta)\,\mathrm{sech}^2(\pi\alpha)}{\tanh^2(\pi\alpha) + \tan^2(\pi\beta)} \tag{4}$$

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Equations (3) and (4) are identical to those given in Ref. 7 which is an analysis restricted to axial instabilities. In Ref. 7 the derivation of the equation for the complex nozzle admittance from which Eq. (3) and (4) may be derived implies that the wave length (λ) may be expressed as $2nL_c$, where n is an integer. The equation for λ is given as

$$\lambda = 2\pi(1 - \bar{M}^2)/(\omega/\bar{c}) \tag{5}$$

which, for $\bar{M}_c^2 << 1$, would be true for a normal axial mode.

EVALUATION OF NOZZLE DAMPING

The equation for nozzle damping as given by Ref. 8 may be written in the form (Ref. 7)

$$2\alpha_n \hat{V}\bar{\rho}\bar{c} = - \int_{\substack{\text{nozzle} \\ \text{entrance}}} dS|P_n|^2[(1 + \bar{M}^2) \, \mathrm{Re}\{y_n\} + \bar{M} + \bar{M}|y_n^2|] \tag{6}$$

where

$$\hat{V} = <\int_V dV\{1/2\bar{\rho}u_1 \cdot u_1 + 1/2 \frac{p_1^2}{\bar{\rho}\bar{c}^2} + \frac{\bar{u} \cdot u_1}{\bar{c}^2} p_1 \}>$$

and for the case of uniform conditions at the nozzle entrance

$$\frac{2\bar{\rho} \, \bar{c}\hat{V}\alpha_n}{S_n|P_n|^2} = -[(1 + \bar{M}^2) \, \mathrm{Re}\{y_n\} + \bar{M} + \bar{M}|y_n|^2] \tag{7}$$

The above equations are based on the analysis of Ref. 8.

Using the equations derived in Ref. 7 the resulting axial nozzle damping equation may be written as

$$\frac{\alpha_n L_c}{\bar{c}} = - \frac{\bar{M} + [(1 + \bar{M}^2)/2] \tanh(2\pi\alpha)}{1 + \bar{M} \tanh(2\pi\alpha)} \tag{8}$$

Equation (8) is not applicable for a submerged nozzle and is <u>most</u> applicable for the use when the parameter σ_n is equal to one.

SHORT NOZZLE THEORY

The method in Ref. 2 and 7 may be used for calculating nozzle admittances for pure axial modes. However, much simpler treatments are available for "short" nozzles for which the nozzle length is much less than the wave length. From Ref. 9 the equation for nozzle damping may be expressed as

$$\alpha_n = \frac{-\overline{Mc}\ \frac{\gamma+1}{2}\left(1 + \frac{\gamma+1}{2}\overline{M}^2\right)}{L_c} \tag{9}$$

which, for $\gamma = 1.4$ and $\overline{M}^2 \ll 1$ yields the result

$$\alpha_n = -1.2\,\frac{\overline{u}}{L_c} \tag{10}$$

Experimental data (Ref. 10) gave the empirical result

$$\alpha_n = -1.0\,\frac{\overline{c}}{L_c}\,J \tag{11}$$

for resonant and decay test results and

$$\alpha_n = -0.3\,\frac{\overline{c}}{L_c}\,J \tag{12}$$

for impedance tube technique. The latter equation was for preliminary results and the discrepancy was not explained.

To compare the calculated equation with the experimental, Zinn (Ref. 9) expressed the Mach number in terms of J and substituted the result into Eq. (9) yielding the approximate equation

$$\alpha_n = -\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma-3}{2(\gamma-1)}}\frac{\overline{c}}{L_c}\,J \tag{13}$$

and for $\gamma = 1.4$, Eq. (12) reduces to

$$\alpha_n = -0.69\,\frac{\overline{c}}{L_c}\,J \tag{14}$$

which falls between the values predicted by Eq. (12) and (13).

For a more rigorous treatment of axial mode nozzle damping the reader is referred to Ref. 7 and 11. The technique employed involves the evaluation of the admittance by an impedance tube technique. The results indicate that the short nozzle theory tends to give somewhat low results (Ref. 11).

The concept of capture area is discussed in Ref. 5 and 9. The capture area is, in general, greater than the entrance area of a nozzle with a cross section less than the port area. In both references it is shown that the measured nozzle admittance will include damping effects attributable to both the nozzle and the nonuniform flow region upstream of the nozzle.

SCALING

Efforts have been made to develop scaling laws from which nozzle admittance measurements on one nozzle may be correlated with the admittance of other nozzles. In brief, for longitudinal modes the admittance depends on a dimensionless frequency ($\omega r_c/\bar{c}$), the axial distribution of the steady state Mach number in the convergent section of the nozzle, and γ (Ref. 12). In what follows, when the performance of two nozzles are compared the values of γ for each gas will be assumed equal. Then y_n for two nozzles with equal entrance Mach numbers will be the same if the nozzle geometries only differ by a linear "contraction" or "stretching" and if the dimensionless frequencies are related by the expression: $F_2 = F_1/\sigma$ in which σ is the ratio (Z_2/Z_1) of a characteristic length.

A second type of scaling is that between cold flow admittance data and actual motor data (Ref. 12).

In order to compare the admittance values it is first necessary that the nozzles be geometrically similar. In addition, the wave lengths must be equal. Then if $M^2 << 1$ (Ref. 12) the nozzle admittance (Y_n) of the actual motor is obtained from the cold flow prototype by the relation

$$(Y_n)_H = \frac{\bar{\rho}_E \bar{c}_E}{\bar{\rho}_H \bar{c}_H} (Y_n)_E \tag{15}$$

In Ref. 12 it is stated that experimental testing is required to establish the validity of the previous correlation.

SUBMERGED NOZZLES AND MULTI NOZZLE CONFIGURATIONS

In many solid propellant rocket motors the nozzle is submerged as shown schematically in Figure 1. Experimental evidence has been obtained that indicates the submerged nozzle provides less damping then does a "conventional" nozzle (Ref. 13).

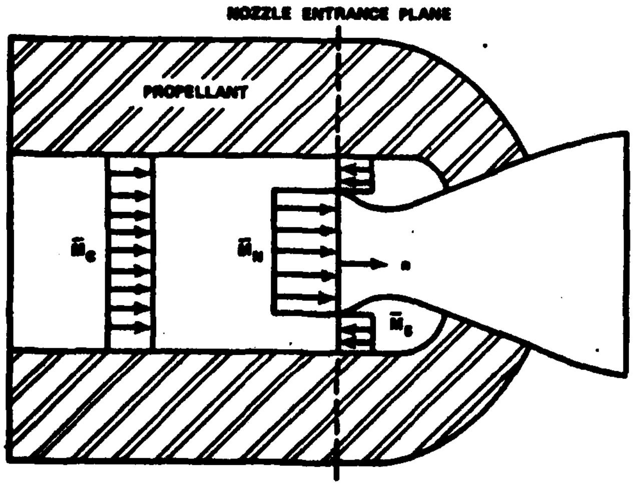

FIGURE 1. Schematic Representation of a Submerged Rocket Nozzle. Chamber cross-sectional area = S_C; cavity entrance area = S_S; nozzle entrance area = S_N.

Equation (6) was used for calculating axial mode damping for the geometry shown in Figure 1 with the exception that Janardan and Zinn (Ref. 13) feel that the integration should be across the entire plane of the nozzle entrance as indicated. Assuming a one-dimensional axial mean flow and wave structure the dimensionless nozzle alpha is then given by Eq. (16)

$$\frac{\alpha_n L_c}{\bar{c}} = - \frac{(\bar{M}_n \sigma_n - \bar{M}_s \sigma_s) + [(1 + \bar{M}_n^2 \sigma_n + \bar{M}_s^2 \sigma_s)/2] \tanh(2\pi\alpha)}{[1 + \bar{M}_c \tanh(2\pi\alpha)]} \tag{16}$$

As pointed out in Ref. 13 when there is no cavity flow the equation reduces to

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$$\frac{\alpha_n L_c}{\bar{c}} = -\frac{\bar{M}_n \sigma_n + [(1 + \bar{M}_n^2 \sigma_n)/2] \tanh(2\pi\alpha)}{[1 + \bar{M}_c \tanh(2\pi\alpha)]} \tag{17}$$

Thus the presence of reverse flow adds two terms of opposite sign. The damping is decreased by the term $\bar{M}_s \sigma_s$ which is larger than the additional damping term $(-1/2\ \bar{M}_s^2\ \sigma_s\ \tanh(2\pi\alpha)$. Thus a decrease of damping is caused by recessing the nozzle. It should be noted that Eq. (16) and (17) reduce to Eq. (8) if the area ratio is set equal to one, a result which should be expected. Equation (16) may be evaluated for a known geometry and fluid properties provided α is determined. In Ref. 13, for the geometry shown schematically in Figure 1 the authors found that the parameter α was nearly independent of cavity depth and cavity flow rate. The value of the parameter α may be determined by the modified impedance tube technique, for example, Ref. 6 and 7. In Ref. 7 it is pointed out that nozzle damping can be effected by the number of ports. They found that quadruple port nozzles provided less damping for axial instabilities then for single and dual ports whose damping was approximately the same.

CONCLUSIONS

The current state of the art precludes the direct calculation of nozzle losses for many common solid propellant motor-nozzle configurations. The calculation for a cylindrical motor terminating with a "conventional" nozzle ($A_c/A_N = 1$) with slowly tapering entrance section may be treated reasonably well for one- or three-dimensional acoustic oscillations in the nozzle. This configuration would apply to many liquid rocket motors but not, in general, to solid propellant motors. Equations and experimental data are available for evaluation of damping of axial modes by short nozzles.

The current available approach for evaluating nozzle damping for motor geometries and wave structure not detailed above is to evaluate the nozzle plane admittance by experimental methods, e.g., impedance tube methods, and use that result to calculate nozzle damping.

The problem of evaluating nozzle damping in solid propellant rockets is a difficult and not fully solved problem. However, nozzle damping may, in many cases, be an important factor when it comes to using stability models as a means of predicting stability trends for motors. In particular, methods (analytical and/or experimental) are needed for determining nozzle damping of transverse and three-dimensional modes of oscillation in solid propellant rocket motors having complex port and nozzle geometries.

NOMENCLATURE

c	Sonic velocity
F	Dimensionless frequency $(\omega r_c/\bar{c})$
J	Area ratio of nozzle throat to nozzle entrance plane
L_c	Chamber length
M	Mach number
P	Pressure amplitude
p_1	Pressure perturbation
r_c	Chamber radius
S	Surface area
S_{mn}	nth root of the equation $\frac{dJ_m(x)}{dx}$
S_n	Nozzle entrance area
$\bar{U}$	Mean velocity
u_1	Velocity perturbation
V	Volume
$\hat{V}$	See Eq. (6)
Y	Nozzle admittance u_1/p_1
y	Non-dimensional admittance $\overline{\rho c}\ (u_1/p_1)$
α	Ratio of amplitude of reflected pressure wave to amplitude of incident wave at the nozzle entrance defined by $e^{-2\pi\alpha}$
α_n	Temporal nozzle decay coefficient
β	Phase change between incident and reflected pressure waves at nozzle entrance defined by $\pi(1 + 2\beta)$
γ	Ratio of specific heats
λ	Wave length
ρ	Density
σ	Ratio of characteristic length of two nozzles
σ_n	S_n/S_c
σ_s	S_s/S_c
ω	Angular frequency

Subscripts

c	Chamber cross-section
i	Imaginary
n	Nozzle entrance cross-section
H	Value for "actual" rocket motor
E	Value for prototype motor
r	Real
s	Area at nozzle entrance plane inclusive of nozzle entrance section
$\overline{(\ )}$	Average value
< >	Time average value over a cycle

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